09441 Abstracts Collection
The Constraint Satisfaction Problem: Complexity and Approximability
— Dagstuhl Seminar —

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Abstract. From 25th to 30th October 2009, the Dagstuhl Seminar 09441 "The Constraint Satisfaction Problem: Complexity and Approximability" was held in Schloss Dagstuhl – Leibniz Center for Informatics. During the seminar, several participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and ideas are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

Keywords. Constraint satisfaction problem (CSP), satisfiability, computational complexity, CSP dichotomy conjecture, hardness of approximation, unique games conjecture, universal algebra, logic

09441 Executive Summary – The Constraint Satisfaction Problem: Complexity and Approximability

The seminar brought together forty researchers from dißerent highly advanced areas of constraint satisfaction and with complementary ex- pertise (logical, algebraic, combinatorial, probabilistic aspects). The list of participants contained both senior and junior researchers and a small number of advanced graduate students.

Keywords: Constraint satisfaction problem (CSP), satisfiability, computational complexity, CSP dichotomy conjecture, hardness of approximation, unique games conjecture, universal algebra, logic

Joint work of: Bulatov, Andrei A.; Grohe, Martin; Kolaitis, Phokion G.; Krokhin, Andrei

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2010/2370

Approximation Resistance

Per Austrin (KTH Stockholm, SE)

It is well-known and easy to see that for any CSP, a random assignment to the variables gives a constant factor approximation algorithm for the problem of maximizing the number of satisfied constraints. A CSP is said to be approximation resistant if it is hard to approximate to within any constant better than the one obtained by the random assignment.

In recent years we have gained a lot of understanding about approximation resistance, but many questions are still open. This talk surveys the known and the unknown in this area, and describes some common proof techniques.

Constraint Satisfaction Problems over the Real Numbers

Manuel Bodirsky (Ecole Polytechnique - Palaiseau, FR)

The feasibility problem for linear programs is a famous constraint satisfaction problem that can be solved in polynomial time.

In this talk we study the question how far the constraint language of all linear inequalities can be expanded such that the corresponding constraint satisfaction problem can still be solved in polynomial time.

We also discuss whether polymorphisms and the universal-algebraic approach can be used to answer this question.

Keywords: Constraint Satisfaction, Semidefinite Programs, Polymorphisms, Real Algebra

Counting constraint satisfaction problems

Andrei A. Bulatov (Simon Fraser University - Burnaby, CA)

The Counting Constraint Satisfaction Problem (#CSP(H)) over a finite relational structure H can be expressed as follows: given a relational structure G over the same vocabulary, determine the number of homomorphisms from G to H. We survey results on the complexity of this problem including a characterization of relational structures H for which #CSP(H) can be solved in polynomial time.

Keywords: Counting problems, CSP, complexity, dichotomy theorem

CSP duality and trees of bounded pathwidth

Catarina Carvalho (University of Lisboa, PT)

A constraint satisfaction problem for a structure B has a duality of some type if the existence of a homomorphism from a given structure A to B is equivalent to the non-existence of a homomorphism to A from a structure belonging to a certain nice class.

We study non-uniform constraint satisfaction problems definable in monadic Datalog stratified by the use of non-linearity. We show how such problems can be described in terms of homomorphism dualities involving trees of bounded pathwidth and in algebraic terms.

This is joint work with V. Dalmau and A. Krokhin.

What is a tractable class? (A tale of width notions and a few shifty characters.)

Hubie Chen (Univ. Pompeu Fabra - Barcelona, ES)

We will engage in a philosophical and technical discussion of the title question, drawing on examples from the study of structurally restricted constraint satisfaction.

Keywords: Tractable class

A Galois connection for Valued Constraint Satisfaction

David Cohen (RHUL - London, GB)

A constraint in classical CSP is a pair (σ, ρ) where σ is a list of variables, from a set V, called the scope and ρ is a relation, over a domain D, whose arity is the size of the scope.

For any relation ρ we have a clone of functions, called the polymorphisms of ρ. We have a well-known and useful Galois connection generated by the binary relation $\{(\rho, f) \mid f \text{ is a polymorphism of } \rho\}$. The two associated mappings are called Inv and Pol. The closed sets of relations are the the sets closed under Primitive Positive definability (CSP expressibility) and the Clones generated by sets of functions.

That is: Inv(Pol($\mathit{\Gamma}$) is the set of relations generated by PP formula from $\mathit{\Gamma}$, whilst Pol(Inv(F)) is the clone generated by F.

Since finitely extending a CSP language by expressible relations does not increase its complexity we have the rather nice result that the complexity of a CSP language is characterized by its clone of polymorphisms.

We have a natural and useful extension (VCSP) to CSP that replaces the relation ρ in a constraint with a function which maps all possible tuples of domain elements to costs in some set (usually the positive rationals with infinity). The VCSP analogue to expressibility uses minimization rather than projection, and addition rather than relational join. It is thus closure under operations of a tropical algebra that corresponds to PP definability.

We showed in 2006 that VCSP expressibility is precisely characterized by the set of fractional polymorphisms. These are multisets of polymorphisms (of the relations obtained from the cost functions by allowing all finite valued tuples and excluding those with infinite values).

The closed sets under the natural Galois connection are called MultiClones and we have the two associated mappings, Imp and MPol with analogous meanings to Inv and Pol. We have therefore shown that Imp(MPol(Γ)) is the set of (VCSP) expressible cost functions for Γ. By similar arguments to those used for classical CSP we have shown that (finitely) extending a VCSP language by expressibility does not increase its complexity, so the multiclone characterizes the complexity of a language of cost functions.

We have now managed to capture an algebraic definition of closure of a set of fractional polymorphisms. That is we have an algebraic characterization of a multiclone. Here we embed fractional polymorphisms into a purely algebraic object generated by extended superpositions and three other simple operations: cancellation, multiplication and division.

The proof is straightforward and uses Farkas' Lemma.

If the clone of polymorphisms has a further property: For every x and k-ary term t with free variables $x, y_1, ..., y_k$ the equation $x = t(x, y_1, ...y_k)$ has a solution. then we can precisely capture the fractional polymorphisms that improve a set of cost functions, rather than embedding them in a larger algebraic object.

That is, the subset of the multiclone consisting precisely of fractional polymorphisms, is itself generated by closure under an algebraic operation. The algebraic closure operation here is (again) just a restricted form of superposition. The proof here uses equational reasoning.

The important class of submodular functions have a multiclone with this property.

It is to be hoped that analysis of MultiClones will be fruitful in understanding the complexity of VCSP languages.

Keywords: Galois Connection, Complexity, VCSP, Clone, Multiclone

Enumerating Homomorphisms

Victor Dalmau (Univ. Pompeu Fabra - Barcelona, ES)

The homomorphism problem for relational structures is an abstract way of formulating constraint satisfaction problems (CSP) and various problems in database theory. The decision version of the homomorphism problem received a lot of attention in literature; in particular, the way the graph-theoretical structure of the variables and constraints influences the complexity of the problem is intensively studied. Here we study the problem of enumerating all the solutions with polynomial delay from a similar point of view. It turns out that the enumeration problem behaves very differently from the decision version. We give evidence that it is unlikely that a characterization result similar to the decision version can be obtained. Nevertheless, we show nontrivial cases where enumeration can be done with polynomial delay.

Joint work of: Bulatov, Andrei; Dalmau, Victor; Grohe, Martin; Marx, Daniel

The Complexity of the List Homomorphism Problem for Graphs

Laszlo Egri (McGill University - Montreal, CA)

We completely characterize the class of graphs for which the list homomorphism problem is in Logspace. Together with previous results, this implies that for every graph H, the list homomorphism problem is either NP-complete, NL-complete, L-complete or has finite duality. The central result is an inductive definition of graphs whose list homomorphism problem is solvable in Logspace. We obtain both algebraic and combinatorial characterizations. A characterization by forbidden (induced) subgraphs is given as well. In particular, the reflexive graphs whose list homomorphism problem is in Logspace are the trivially perfect graphs, or equivalently the graphs that contain no induced path of length three and no cycle of length four. In the irreflexive case, an analogous result is obtained: those with a list homomorphism problem in Logspace are the bipartite graphs that contain no induced path of length five and no cycle of length six. The general case can also be characterized by a set of forbidden subgraphs.

Keywords: Constraint satisfaction problem, complexity, list-homomorphism

Joint work of: Egri, Laszlo; Krokhin, Andrei; Larose, Benoit; Tesson, Pascal

The Complexity of Approximating Boolean #CSP

Leslie Ann Goldberg (University of Liverpool, GB)

Let Γ be a Boolean constraint language.

Creignou and Hermann gave a dichotomy theorem for the problem #CSP(Γ) which is the problem of counting the number of satisfying assignments to a CSP instance with constraint language Γ.

They showed that if every relation in Γ is affine then #CSP(Γ) is solvable in polynomial time. Otherwise, it is #P-complete.

This talk considers the problem of approximating #CSP(Γ).

I'll start by filling in some background on the complexity of approximating #P problems — this is based on joint work with Dyer, Greenhill, and Jerrum.

This will allow me to describe an approximation trichotomy for Boolean #CSP which is joint work with Dyer and Jerrum.

We showed that three things can happen.

Either every relation in Γ is affine, in which case #CSP(Γ) is polynomial-time solvable as noted above.

Otherwise, if every relation in Γ is in a certain co-clone in Post's lattice (the co-clone IM_2) then the approximation problem #CSP(Γ) is complete with respect to approximation-preserving reductions in a logically-defined subclass of #P. In the remaining case, we show that there is no FPRAS for #CSP(Γ) unless NP=RP.

If time permits, I will describe some newer extensions to the bounded-degree case. These are joint work with Dyer, Jalsenius and Richerby.

Keywords: Complexity, counting, constraint satisfaction

Approximability of CSPs (A tutorial)

Venkatesan Guruswami (Carnegie Mellon University - Pittsburgh, US)

This talk is intended to be a tutorial on the approximability of constraint satisfaction problems, where the goal is to find an assignment satisfying as many constraints as possible. After a brief discussion of some classification/dichotomy results on Max CSP, we will focus on some salient results in the program of understanding the approximability of a CSP, both from the algorithmic side and the hardness side.

On the algorithmic side, linear and semidefinite programming relaxations have been the most successful tool for designing good approximation algorithms.

We will discuss some examples of such algorithms and the non-trivial approximation ratios they are able to guarantee.

On the hardness side, we will review some tight non-approximability results obtained by reductions from a CSP called Label Cover, using the methodology of Long Code/Dictatorship testing. These results prove that many natural CSPs are approximation resistant, in that outputting a random assignment yields essentially the best possible approximation guarantee.

We will then discuss recent progress based on the Unique Games conjecture, which is able to identify the approximation threshold of every CSP as the integrality gap of a natural semidefinite programming relaxation.

Trichotomy in the Complexity of Minimal Inference

Miki Hermann (Ecole Polytechnique - Palaiseau, FR)

We study the complexity of the propositional minimal inference problem. Its complexity has been extensively studied before because of its fundamental importance in artificial intelligence and nonmonotonic logics.

We prove that the complexity of the minimal inference problem with unbounded queries has a trichotomy (between P, coNP-complete, and Π_2^{P}-complete). This result finally settles with a positive answer the trichotomy conjecture of Kirousis and Kolaitis [A dichotomy in the complexity of propositional circumscription, LICS'01] in the unbounded case. We also present simple and efficiently computable criteria separating the different cases.

Full Paper:
http://dx.doi.org/10.1109/LICS.2009.14

A complexity dichotomy for hypergraph partition functions

Mark Jerrum (University of London, GB)

We consider the complexity of counting homomorphisms from an r-uniform hypergraph G to a symmetric r-ary relation H. We give a dichotomy theorem for $r > 2$, showing for which H this problem is in FP and for which H it is #P-complete. This generalises a theorem of Dyer and Greenhill (2000) for the case $r = 2$, which corresponds to counting graph homomorphisms. Our dichotomy theorem extends to the case in which the relation H is weighted, and the goal is to compute the *partition function*, which is the sum of weights of the homomorphisms. This problem is motivated by statistical physics, where it arises as computing the partition function for particle models in which certain combinations of r sites interact symmetrically. In the weighted case, our dichotomy theorem generalises a result of Bulatov and Grohe (2005) for graphs, where $r = 2$. When $r = 2$, the polynomial time cases of the dichotomy correspond simply to rank-1 weights. Surprisingly, for all $r > 2$ the polynomial time cases of the dichotomy have rather more structure. It turns out that the weights must be superimposed on a combinatorial structure defined by solutions of an equation over an Abelian group. Our result also gives a dichotomy for a closely related constraint satisfaction problem.

Keywords: Computational complexity, Constraint satisfaction problems, Counting problems, Graph homomorphisms, Hypergraphs, Partition functions

Joint work of: Dyer, Martin; Goldberg, Leslie Ann; Jerrum, Mark

Full Paper:
http://arxiv.org/abs/0811.0037

Coloring Random graphs: A Short Survey and Biased Survey

Lefteris M. Kirousis (RACTI - Rion, GR)

We will shortly present some recent results concerning the chromatic number of random graphs.

The setting is as follows: We consider a probability space with graphs of a given average degree d and n vertices. In the first case (the Erdős-Rényi graphs), the probability space comprises all graphs with average degree d and n vertices (here "average" means the sum of the degrees of all vertices divided by n). In the second case (regular graphs), the space comprises of all graphs where all n vertices have the same degree d. The probability measure in both cases is uniform.

In both cases the chromatic number exhibits interesting threshold behavior: for a given average degree (given constant degree, for the regular case, respectively), the chromatic number of (asymptotically with n) almost all graphs lies within a common, small window of 1–3 integers. However as the degree increases, at specific values, this window undergoes abrupt changes.

Keywords: Random Graphs, Chromatic Number

Random Graphs and the Parity Quantifier

Swastik Kopparty (MIT - Cambridge, US)

The classical zero-one law for first-order logic on random graphs says that for every first-order property φ in the theory of graphs and every $p \in (0,1)$, the probability that the random graph $G(n,p)$ satisfies φ approaches either 0 or 1 as n approaches infinity.

It is well known that this law fails to hold for any formalism that can express the parity quantifier: for certain properties, the probability that $G(n,p)$ satisfies the property need not converge, and for others the limit may be strictly between 0 and 1.

In this work, we capture the limiting behavior of properties definable in first order logic augmented with the parity quantifier, $FO[parity]$, over $G(n,p)$, thus eluding the above hurdles. Specifically, we establish the following "modular convergence law":

For every $FO[parity]$ sentence φ, there are two explicitly computable rational numbers a_0, a_1, such that for $i \in \{0,1\}$, as n approaches infinity, the probability that the random graph $G(2n+i,p)$ satisfies φ approaches a_i.

Our results also extend appropriately to FO equipped with Mod_q quantifiers for prime q.

In the process of deriving the above theorem, we explore a new question that may be of interest in its own right. Specifically, we study the joint distribution of the subgraph statistics modulo 2 of $G(n,p)$: namely, the number of copies, mod 2, of a fixed number of graphs $F_1, \ldots, F_\ell$ of bounded size in $G(n,p)$. We first show that every $FO[parity]$ property φ is almost surely determined by subgraph statistics modulo 2 of the above type.

Next, we show that the limiting joint distribution of the subgraph statistics modulo 2 depends only on $n \mod 2$, and we determine this limiting distribution completely. Interestingly, both these steps are based on a common technique using multivariate polynomials over finite fields and, in particular, on a new generalization of the Gowers norm.

The first step above is analogous to the Razborov-Smolensky method for lower bounds for AC_0 with parity gates, yet stronger in certain ways. For instance, it allows us to obtain examples of simple graph properties that are exponentially uncorrelated with every $FO[parity]$ sentence, which is something that is not known for $AC_0[parity]$.

Keywords: 0-1 law, low-degree polynomials, Gowers norm, Razborov-Smolensky

Joint work of: Kolaitis, Phokion; Kopparty, Swastik

CSPs of bounded width

Marcin Kozik (Jagiellonian University - Kraków, PL)

Constraint Satisfaction Problem has bounded width if it can be solved by local consistency checking. During the talk I will introduce the local consistency checking algorithm, the notion of strategy and present an overview of the proof of the algebraic characterization for this group of CSPs.

Keywords: CSP, local consistency checking, bounded width

Joint work of: Barto, Libor; Kozik, Marcin

The algebraic and logical approaches to the CSP: A tutorial

Andrei Krokhin (University of Durham, GB)

In this tutorial, I will demonstrate how to translate questions about the computational complexity of the CSP with a given constraint language (i.e. set of allowed constraint relations/predicates) into questions about algebras. I will show how universal-algebraic and logical techniques can give a deep insight into the computational structure of the CSP. Time permitting, short proofs of some celebrated computational results will be given.

Keywords: CSP, universal algebra, logic

Submodular functions on diamonds and Max CSP

Fredrik Kuivinen (Linköping University, SE)

In this talk we will look at Max CSP parameterised by the allowed set of predicates.

In particular, we will be interested in which sets of predicates that give rise to Max CSPs which can be solved to optimality in polynomial time.

The known tractable cases for this problem are closely connected to the submodular function minimisation problem (SFM) over finite lattices. New tractability results for SFM implies new tractability results for Max CSP. It has been conjectured that SFM is the only reason for Max CSP to be tractable, but no one has yet managed to prove this.

For some finite lattices there are efficient algorithms for the SFM problem, but not for all. I will present a new algorithm for the SFM problem when the lattice is a diamond (the five element modular non-distributive lattice). I will also mention some connections to the Max CSP-problem.

The complexity of positive first-order logic without equality II: The four-element case

Barnaby Martin (University VII - Paris, FR)

We study the complexity of evaluating positive equality-free sentences of first-order (FO) logic over a fixed, finite structure $\mathcal{B}$. This may be seen as a natural generalisation of the non-uniform quantified constraint satisfaction problem QCSP($\mathcal{B}$). We introduce surjective hyper-endomorphisms and use them in proving a Galois connection that characterises definability in positive equality-free FO. Through an algebraic method, we derive a complete complexity classification for our problems as $\mathcal{B}$ ranges over structures of size at most three. Specifically, each problem is either in Logspace, is NP-complete, is coNP-complete or is Pspace-complete.

Keywords: Constraint Satisfaction

Joint work of: Martin, Barnaby; Madelaine, Florent

Full Paper:
http://www2.computer.org/portal/web/csdl/doi/10.1109/LICS.2009.15

Structural complexity of CSPs: the role of treewidth and its generalizations

Daniel Marx (Tel Aviv University, IL)

An important direction in the theoretical research on CSP is understanding how the complexity of the problem is influenced by the way the constraints connect the variables. For example, for binary constraint satisfaction problems, it is well-known that the problem can be solved in polynomial time if the primal graph of the instance has bounded treewidth, and complexity results show that no other graph property guarantees polynomial-time solvability. The situation is quite different when constraints of unbounded arity are allowed and especially for the instances arising from database-theoretic applications; in this case, hypergraph notions such us hypertree width become relevant for the complexity of the problem. The talk will survey the known results in this direction and report on some recent developments.

Beyond fractional hypertree width

Daniel Marx (Tel Aviv University, IL)

Constraint satisfaction is known to be polynomial-time solvable when restricted to a set of hypergraphs with bounded fractional hypertree width. We introduce a new width measure "submodular width" and show that constraint satisfaction restricted to any set of hypergraphs with bounded submodular width is fixed-parameter tractable parameterized by treewidth. Conversely, we show that if constraint satisfaction is restricted to an arbitrary set of hypergraphs with unbounded submodular width, then the problem is not fixed-paramegter tractable (under suitable complexity assumptions).

Complexity of Approximating Constraint Satisfaction Problems

Prasad Raghavendra (University of Washington - Seattle, US)

A Max-CSP such as MaxCut, Max3-SAT is specified by a set of relations/predicates over a fixed domain. An instance of the problem consists of the predicates applied over subsets of a set of variables. The goal is to find an assignment that satisfies the maximum number of the predicates.

In this work, we show that a simple semidefinite program yields the optimal approximation ratio for every Max-CSP problem under the Unique Games Conjecture.

At the heart of the proof, lies a generic algorithm for Max-CSP problems. This generic algorithm obtains an α-approximation for a CSP, only using the fact that there are certain α-approximate polymorphisms for it.

Full Paper:
http://www.cs.washington.edu/homes/prasad/Files/extabstract.pdf

A dichotomy theorem for the general minimum cost homomorphism problem

Rustem Takhanov (Linköping University, SE)

In the constraint satisfaction problem (CSP), the aim is to find an assignment of values to a set of variables subject to specified constraints.

In the minimum cost homomorphism problem ($MinHom$), one is additionally given weights c_{va} for every variable v and value a, and the aim is to find an assignment f to the variables that minimizes $\sum_v c_{vf(v)}$. Let $MinHom\,(\Gamma)$ denote the $MinHom$ problem parameterized by the set of predicates allowed for constraints. $MinHom\,(\Gamma)$ is related to many well-studied combinatorial optimization problems, and concrete applications can be found in, for instance, defence logistics and machine learning. We show that $MinHom\,(\Gamma)$ can be studied by using algebraic methods similar to those used for CSPs.

With the aid of algebraic techniques, we classify the computational complexity of $MinHom\,(\Gamma)$ for all choices of Γ.

Keywords: Minimum cost homomorphisms problem, relational clones, constraint satisfaction problem, perfect graphs, supervised learning

Partition functions with mixed signs and complex weights

Marc Thurley (HU Berlin, DE)

Partition functions are weighted generalizations of graph homomorphism counting problems.

Every partition function is a graph invariant defined by some quadratic matrix A.

The complexity of computing these partition functions has been studied by Dyer and Greenhill for symmetric 0-1 matrices. They obtained a dichotomy by showing that for every such matrix the associated partition function can either be computed in PTIME or is #P-hard to compute.

This has been generalized by Bulatov and Grohe (2005) to symmetric matrices with arbitrary non-negative entries.

In my talk, I will focus on further extensions of these results. Together with Leslie Goldberg, Martin Grohe, and Mark Jerrum, we have developed a dichotomy result which captures the cases of negative entries. This reveals surprising connections between partition functions and the problem of computing the number of roots of multivariate GF(2) polynomials.

Further, I will dicuss two cases of partition functions defined by matrices with complex valued entries. The first is a dichotomy for partition functions on complex symmetric matrices by Cai, Chen and Lu. The second one is that of partition functions on Hermitian matrices which I studied.

On the complexity of equivalence and isomorphism of primitive positive formulas

Matt Valeriote (McMaster University - Hamilton, CA)

A primitive positive formula is a first-order formula defined from atomic formulas and equality of variables using conjunction and existential quantification.

In this talk, we study the complexity of two basic questions associated to primitive positive formulas:

(1) Given two such formulas having the same free variables and a relational structure, are the formulas equivalent over the structure? That is, do they admit the same satisfying assignments?

(2) Given two such formulas and a relational structure, are the formulas isomorphic over the structure? By isomorphic, we mean that there is a way to rename the free variables of one formula with the free variables of the other so that the formulas are equivalent.

We study both of these problems with respect to various fixed structures.

That is, we parameterize each of these problems with respect to the structure to obtain a family of problems, containing one member for each structure, and study the resulting two families of problems.

As has been shown for the Constraint Satisfaction Problem, we demonstrate that to each structure one can associate an algebra in such a way that the complexity of all structures associated to the same algebra is the same. We present a number of sufficient conditions for hardness for various complexity classes, as well as sufficient conditions for containment in various complexity classes.

The presented work is joint with Simone Bova (Vanderbilt University) and Hubie Chen (Universitat Pompeu Fabra).

On the Expression Complexity of Equivalence and Isomorphism of Primitive Positive Formulas

Matt Valeriote (McMaster University - Hamilton, CA)

We study the complexity of equivalence and isomorphism on primitive positive formulas with respect to a given structure. We study these problems for various fixed structures; we present generic hardness and complexity class containment results, and give classification theorems for the case of two-element (boolean) structures.

Keywords: Expression complexity, equivalence, isomorphism, primitive positive formulas

Joint work of: Valeriote, Matt; Bova, Simone; Chen, Hubie

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2010/2369

Kernelizability of boolean Min Ones CSPs

Magnus Wahlstroem (MPI für Informatik - Saarbrücken, DE)

We give a dichotomy for the existence of polynomial kernelizations (under the standard parameterization) for the Min Ones CSP(Γ) problems, for finite boolean constraint languages Γ.

In parameterized complexity, each problem instance comes with a parameter k; a typical example could be an optimization problem given as (I, k), where the question is whether I has a solution of cost at most k (and this is the setting used in this talk). We say that the problem is parameterized.

A kernelization algorithm is a type of polynomial time pre-processing on parameterized problems: Given an input (I, k), a kernelization reduces the problem in polynomial time to an equivalent instance (I', k'), where the size of I' and k' are both bounded by a function of k alone (and typically $k' \leq k$). This reduced instance is referred to as a kernel. It is an important question in parameterized complexity what the best possible size bounds on problem kernels are, and in particular which problems admit a polynomial kernelization (that is, a kernelization where the kernel size is polynomial in k). Recently developed methods (Fortnow and Santhanam, STOC 2008; Bodlaender et al., ICALP 2008) provide a framework for excluding polynomial kernelizations for certain problems (under standard complexity theoretical assumptions), allowing us to settle these questions in both directions.

Using this framework, we give a characterization that decides the existence of polynomial kernelizations for boolean Min Ones CSP problems, for finite constraint languages Γ. Specifically, for the positive cases we provide a kernelization to a kernel of $O(k^{d+1})$ variables, where d is the maximum arity of a relation in Γ, while we show that for all remaining cases, under standard complexity theoretical assumptions, no polynomial kernelizations are possible.

Joint work of: Kratsch, Stefan; Wahlstroem, Magnus

PP-DEFINABILITY IS CO-NEXPTIME-COMPLETE

Ross Willard (University of Waterloo, CA)

$\exists$-InvSat is the problem which takes as input a relation R and a finite set $\mathcal{S}$ of relations on the same finite domain D, and asks whether R is definable by a conjunctive query over $\mathcal{S}$, i.e., by a formula of the form $\exists \mathbf{y} \varphi(\mathbf{x}, \mathbf{y})$ where φ is a conjunction of atomic formulas built on the relations in $\mathcal{S} \cup \{=\}$. (These are also called *primitive positive formulas.*) The problem is known to be in co-NExpTime, and has been shown to be tractable on the boolean domain.

We show that there exists $k > 2$ such that $\exists$-InvSat is co-NExpTime complete on k-element domains, answering a question of Creignou, Kolaitis and Zanuttini.

Keywords: Conjunctive query, primitive positive, structure identification, existential inverse satisfiability

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2010/2368